Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name : Engineering Mathematics – IV

Subject Code : 4TE04EMT1 Branch: B.Tech (Auto, Mech, Civil, EE, EC)

Semester : 4 Date : 24/04/2018 Time : 10:30 To 01:30 Marks : 70

Instructions:

1. Use of Programmable calculator & any other electronic instrument is prohibited.
2. Instructions written on main answer book are strictly to be obeyed.
3. Draw neat diagrams and figures (if necessary) at right places.
4. Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) E^{-1} equal to

(A) $1-\nabla$ (B) $1+\nabla$ (C) $1+\delta$ (D) $1-\delta$

b) hD equal to

(A) $\log\left(1+\Delta\right)$ (B) $\log\left(1-\Delta\right)$ (C) $\log\left(1+\mathrm{E}\right)$ (D) $\log\left(1-\mathrm{E}\right)$

c) While evaluating a definite integral by Trapezoidal rule, the accuracy can be increased by taking

(A) small number of sub – intervals (B) large number of sub – intervals

(C) odd number of sub – intervals (D) none of these

d) In application of Simpson's $\frac{1}{3}$ rule, the interval of integration for closer approximation should be

(A) odd and small (B) even and small (C) even and large (D) none of these

e) The Gauss – Jordan method in which the set of equations are transformed into diagonal matrix form.

(A) True (B) False

f) The convergence in the Gauss – Seidel method is faster than Gauss – Jacobi method.

(A) True (B) False

g) The auxiliary quantity k_1 obtained by Runge – Kutta fourth order for the differential equation $\frac{dy}{dx}=x^2+y^2$, $y(0)=1$, when $h=0.1$ is

(A) 0.1 (B) 0 (C) 1 (D) none of these

h) The first approximation y_1 of the initial value problem $\frac{dy}{dx}=x^2+y^2$, $y(0)=0$ obtain by Picard's method is

(A) x^2 (B) $\frac{x^2}{2}$ (C) $\frac{x^3}{3}$ (D) none of these

i) The Fourier sine transform of $f(x)=\begin{cases}k, & 0<x<a\\ 0, & x>a\end{cases}$ is

(A) $\sqrt{\frac{2}{\pi}}k\left(\frac{\sin a\lambda}{\lambda}\right)$ (B) $\sqrt{\frac{2}{\pi}}k\left(\frac{1-\cos a\lambda}{\lambda}\right)$ (C) $\sqrt{\frac{2}{\pi}}k\left(\frac{\sin a\lambda}{a}\right)$

(D) none of these

j) The finite Fourier cosine transform of $f(x)=2x,\ 0<x<4$ is

(A) $\frac{32}{n^2\pi^2}\left[(-1)^n-1\right]$ (B) $\frac{16}{n^2\pi^2}\left[(-1)^n-1\right]$ (C) $\frac{32}{n^2\pi^2}(-1)^n$ (D) none of these

k) Which one of the following is an analytic function

(A) $\mathrm{f}(z)=\mathrm{R}iz$ (B) $\mathrm{f}(z)=\operatorname{Im} z$ (C) $\mathrm{f}(z)=\bar{z}$ (D) $\mathrm{f}(z)=\sin z$

l) The image of circle $|z-1|=1$ in the complex plane, under the mapping $w=\frac{1}{z}$ is

(A) $|w-1|=1$ (B) $u^2+v^2=1$ (C) $v=\frac{1}{z}$ (D) $u=\frac{1}{z}$

m) The magnitude of acceleration vector at $t=0$ on the curve $x=2\cos 3t, y=2\sin 3t$, $z=3t$ is

(A) 6 (B) 9 (C) 18 (D) 3

n) If $\phi=xyz$, the value of $|\text{grad } \phi|$ at the point $(1,2,-1)$ is

(A) 0 (B) 1 (C) 2 (D) 3

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Given **(5)**

x:	10	20	30	40	50
y:	600	512	439	346	243

Using Stiring's formula find y_{35} .

b) Given that **(5)**

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
y	1.00000	1.02470	1.04881	1.07238	1.09544	1.11803	1.14017

Find $\frac{d^2y}{dx^2}$ at $x=1.05$.

c) Find the finite Fourier cosine transform of $f(x)=2x,\quad 0<x<4$. **(4)**

Q-3 **Attempt all questions** **(14)**

a) Solve the following system of equations by Gauss-Seidal method. **(5)**

$10x_1+x_2+2x_3=44,\ 2x_1+10x_2+x_3=51,\ x_1+2x_2+10x_3=61$

b) Using Newton's forward interpolation formula, find the value of $y(2.35)$ if **(5)**

x	2.00	2.25	2.50	2.75	3.00
$f(x)$	9.00	10.06	11.25	12.56	14.00

c) If $f(z)=u+iv$ is an analytic function of z and $u+v=e^x(\cos y+\sin y)$, find $f(z)$. **(4)**

Q-4 **Attempt all questions** **(14)**

a) Use the fourth – order Runge Kutta method to solve $\frac{dy}{dx} = x^2 + y^2; \quad y(0) = 1$. Evaluate the value of y when $x = 0.1$. **(5)**

b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's 3/8th rule. **(5)**

c) Solve the following system of equations by Gauss Elimination Method: **(4)**
$5x - 2y + 3z = 18,\ x + 7y - 3z = -22,\ 2x - y + 6z = 22$

Q-5 **Attempt all questions** **(14)**

a) Show that the function defined by the equation **(5)**

$$\mathrm{f}(z) = \begin{cases} u(x,y) + iv(x,y), & \text{if } z \neq 0 \\ 0 \quad , & \text{if } z = 0 \end{cases}$$

where $u(x,y) = \frac{x^3 - y^3}{x^2 + y^2}$ and $v(x,y) = \frac{x^3 + y^3}{x^2 + y^2}$ is not analytic at $z = 0$ although Cauchy – Riemann equations are satisfied at that poiut.

b) If $\vec{F} = \left(2x^2 - 4z\right)i - 2xyj - 8x^2k$, then evaluate $\iiint_V div\ \vec{F}\ dV$, where V is bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y + z = 1$. **(5)**

c) Given that table of values as **(4)**

x	20	25	30	35
y	0.342	0.423	0.500	0.650

Find $x\left(0.390\right)$ using Lagrange's inverse interpolation formula.

Q-6 **Attempt all questions** **(14)**

a) Prove that $\vec{F} = \left(y\cos z - \sin x\right)i + \left(x\sin z + 2yz\right)j + \left(xy\cos z + y^2\right)k$ is irrotational and find its scalar potential. **(5)**

b) Find the bilinear transformation which sends the points $z = 0,1,\infty$ into the points $w = -5,-1,3$ respectively. What are the invariant points of the transformation? **(5)**

c) Obtain Picard's second approximation solution of the initial value problem $\frac{dy}{dx} = x^2 + y^2$ for $x = 0.4$ correct to four decimal places, given that $y\left(0\right) = 0$. **(4)**

Q-7 **Attempt all questions** **(14)**

a) Using Cauchy – Riemann equations, prove that if $f\left(z\right) = u + iv$ is analytic with constant modulus, then u, v are constants. **(5)**

b) Using Green's Theorem, evaluate $\oint_C \left[\left(y - \sin x\right)dx + \cos x dy\right]$ where C is the plane triangle enclosed by the lines $y = 0,\ x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$. **(5)**

c) The function $f\left(x\right)$ is given as follows: **(4)**

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y	1	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0

Compute the integral of $f\left(x\right)$ between $x = 0$ and $x = 1.0$ using Trapezoidal rule.

Q-8 **Attempt all questions** **(14)**

a) Use Euler's method to find an approximate value of y at $x = 0.1$, in five steps, given that $\frac{dy}{dx} = x - y^2$ and $y(0) = 1$. **(5)**

b) Find the Fourier sine transform of $f(x) = \begin{cases} 0 & 0 < x < a \\ x & a \le x \le b \\ 0 & x > b \end{cases}$. **(5)**

c) Find the angle between the tangents to the curve $x = t^2 + 1, y = 4t - 3, z = 2t^2 - 6t$ at the points $t = 1$ and $t = 2$. **(4)**